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Sufficient Conditions for Global Asymptotic Stability of delayed Cellular Neural Networks

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ABSTRACT

This paper investigates the global asymptotic stability for the cellular neural networks with time-varying delays. By using the Lyapunov functional method and Razumikhin approach, some sufficient conditions are obtained without demanding the differentiability of the activation function. Moreover, two examples are demonstrated to illustrate the effectiveness of the proposed criterion.

Keywords: *Cellular neural system, Global asymptotic stability, Lyapunov functional, time-varying delay.*

INTRODUCTION

The cellular neural networks have important applications in various areas such as classification, associative memory, parallel computing, especially in solving some optimization problems [1]. In these applications, it is very important to study the stability of neural networks.

In addition, time-delays are unavoidably encountered in the implementation of neural networks, and may cause undesirable dynamic network behaviors such as oscillation and instability. As a consequence, many researchers have focused their attention on the study of stability of the neural networks with delays. In Recent years, some sufficient conditions were presented to ensure the global asymptotic stability (GAS) of the delayed neural networks [2-13]. But in [4-8], the activation functions are assumed to be differentiable. However, in practice the activation functions are not always to be differentiable. In addition, the authors in [2] have studied the following cellular neural networks with the constant delays and the GAS conditions are obtained by employing non smooth analysis. But in practice, time delay is usual time-varying, which can even largely change the dynamics of system in some cases. Therefore, their methods have a conservatism which can be improved upon

$$\dot{x}_i(t) = -x_i(t) + \sum_{j=1}^{n} a_{ij} f_j(t) + \sum_{j=1}^{n} b_{ij} f_j(t-\tau) + I_i,\quad i = 1, 2, ..., n, \qquad (1)$$

n this paper, by using a new method based on the non smooth analysis, we obtain an improved sufficient condition for the GAS of the equilibrium point without demanding the boundedness and differentiability of activation functions. Two examples are provided to show the effectiveness and the benefits of the proposed method.

Notation: Throughout this paper, we use $\lambda_{\min}(\cdot)$ and $\lambda_{\max}(\cdot)$ to denote the minimum and maximum eigenvalue of a real symmetric matrix, respectively. The superscript T represents the transpose. R^n is the n-dimensional Euclidean space. Let $\mu_2(A) = \frac{1}{2}\lambda_{\max}(A + A^T)$, i.e., $\mu_2(A)$ is the largest eigenvalue of the symmetric part of A. It is well known that $\mu_2(A) \le \|A\|_2$. For $x = (x_1, x_2, \cdots, x_n)^T \in R^n$, we denote $|x| = (|x_1|, |x_2|, \cdots, |x_n|)^T$ and

$\mathrm{sgn}(x) = diag(\mathrm{sgn}(x_1), \mathrm{sgn}(x_2), \cdots, \mathrm{sgn}(x_n))^T$, where $\mathrm{sgn}(x) = \begin{cases} -1, \; if \; x_i < 0 \\ 0, if \; x_i < 0 \\ 1, if \; x_i < 0 \end{cases}$.

Problem Statement

Consider the following delay cellular neural networks (DCNN):

$$\dot{x}(t) = -x_{ij}(t) + \sum_{c(k,l)\in N_r(i,j)} A^0_{ij,kl} y_{kl}(t) + \sum_{c(k,l)\in N_r(i,j)} A^1_{ij,kl} y_{kl}(t-\tau(t)) + \sum_{c(k,l)\in N_r(i,j)} B_{ij,kl} U_{kl}(t) + I_{ij} \quad i = 1.2, \cdots, n, \tag{2}$$

where $N_r(i,j)$ denotes r field of $C(i,j)$; x_{ij}, y_{ij}, u_{ij} are the state, output, and input of the $C(i,j)$, respectively. $A^m_{ij,kl}\,(m = 0,1)$ and $B_{ij,kl}$ represent the influence strength of cell $C(k,l)$ output and input on cell $C(i,j)$, respectively. I_{ij} Represents a current bias of cell $C(i,j)$.There are delays $\tau(t) > 0$ between the cells, where $\tau(t)$ is a time varying delay and $|\dot{\tau}(t)| < \tau, 0 < \tau < 1$ .Assume that $B_{ij,kl} = 0$. Let $P = M \times N$, then (2) can be written

$$\dot{x}_i(t) = -x_i(t) + \sum_{j=1}^{P} a_{ij} y_j(t) + \sum_{j=1}^{P} b_{ij} y_j(t-\tau(t)) + I_i,$$

$$i = 1, 2, \cdots, p. \tag{3}$$

or in vector-matrix form

$$\dot{x} = -x + Ay(t) + By(t-\tau(t)) + I. \tag{4}$$

Usually, the relation between the cell output and the cell state satisfies the piecewise linear function (PWL):

$$y_i = f_i(x_i) = \left(\frac{1}{2}\right)(|x_i + 1| - |x_i - 1|). \tag{5}$$

In order to establish the stability conditions for system (3), we first give some usual assumptions and lemmas.

(M_1) The activation function $y_i = f_i(x_i)$ is global Lipschitz continuous in R, i.e. there exists a constant $M_i > 0$ for any $x_1, x_2 \in R$ such that

$$|f_i(x_1) - f_i(x_2)| \le M_i |x_1 - x_2|, i = 1, 2, ..., n.$$

(M_2) $y_i = f_i(x_i)$ is monotonically non decreasing and bounded in R.

Lemma 1[1] Let $A = (a_{ij})$ be an $n \times n$ matrix with non-positive off-diagonal elements. Then each of the following conditions is equivalent to the statement 'A is a non-singular M-matrix'.

F1: All principal minors of A are positive.

F2: All elements of A^{-1} are non-negative.

F3: All diagonal elements of A are positive and there exists a positive diagonal matrix $P = diag\{p_1, p_2, ..., p_n\}$ such that AP is strictly diagonally row-dominant; that is

$$a_{ii} p_i > \sum_{j=1, j\neq i}^{n} p_j |a_{ij}|, \; j = 1, 2, ..., n.$$

F4: All diagonal elements of A are positive and there exists a positive diagonal matrix $P = diag\{p_1, p_2, ..., p_n\}$, such that AP is strictly diagonally column-dominant; that is

$$a_{jj} p_j > \sum_{j=1, i\neq j}^{n} p_i |a_{ij}|, \; j = 1, 2, ..., n.$$

Lemma 2[1] If A and $B = (b_{ij} | b_{ij} \le 0, i \ne j) \ge A$ are M-matrix, then B is also M-matrix M- matrix.

Next, let $x^* = (x_1^*, x_2^*, ..., x_n^*)$ be an equilibrium point of (3) and

$z = x - x^*, g_i(z_i) = f_i(z_i + x_i^*) - f_i(x_i^*), i = 1, 2, ..., p$. From (3) and replaying $x_i(t)$ in (3) by $z_i(t) + x_i^*$, it is not difficult to obtain

$$\dot{z}_i(t) = -z_i(t) + \sum_{j=1}^{P} a_{ij} g_j(z_j) + \sum_{j=1}^{P} b_{ij} g_j(z_j(t-\tau(t))(1-\dot{\tau}(t))),$$

$$i = 1, 2, \cdots, p. \tag{6}$$

Results and Discussion

In this section, we present the new result for the GAS of the equilibrium point of (6).

Theorem 1. Suppose that

$$\max_{1\le j\le P}\left[M_j\left(\sum_{i=1}^{P}|b_{ij}|(1-\tau)+a_{ij}+\sum_{1\le i\ne j\le P}|a_{ij}|\right)\right]<1$$

is satisfied. Then, the unique equilibrium point x^* of system (6) is global asymptotic stability.

Proof. Let

$z(t)\ne 0$ Obviously, $V(z_t)$ is positive definite and radially unbounded.

By calculating the time derivative of $V(z_t)$ along the trajectories of the system (6), we obtain

$$\begin{aligned}\dot V(z_t)\big|_{(6)} &= \sum_{i=1}^{P}\left(M_i\sum_{j=1}^{P}|b_{ji}|-1\right)|z_i|+\sum_{i,j=1}^{P}\left(\operatorname{sgn}(z_i)a_{ij}g_j(z_j)-b_{ij}g_j\left(z_j(t-\tau(t))(1-\dot\tau(t))\right)\operatorname{sgn}(z_i)\right)\\
&-\sum_{i=1}^{P}\left(M_i\sum_{j=1}^{P}|b_{ji}|\right)\left|z_i(t-\tau(t))(1-\dot\tau(t))\right|\\
&\le\sum_{i=1}^{P}\left(M_i\sum_{j=1}^{P}|b_{ji}|-1\right)|z_i|+\sum_{j=1}^{P}\left[\operatorname{sgn}(z_j)a_{jj}g_j(z_j)+\sum_{1\le i\ne j\le P}\operatorname{sgn}(z_j)a_{ij}g_j(z_j)\right]\\
&+\sum_{i,j=1}^{P}b_{ij}g_j\left(z_i(t-\tau(t))(1-\dot\tau(t))\operatorname{sgn}(z_i)\right)-\sum_{i=1}^{P}\left(M_i\sum_{j=1}^{P}|b_{ji}|\right)\left|z_i(t-\tau(t))(1-\dot\tau(t))\right|\\
&\le\sum_{i=1}^{P}\left(M_i\sum_{j=1}^{P}|b_{ji}|-1\right)|z_i|+\sum_{j=1}^{P}\left[\operatorname{sgn}(z_j)a_{jj}g_j(z_j)+\sum_{1\le i\ne j\le P}\operatorname{sgn}(z_j)a_{ij}g_j(z_j)\right]\\
&+\sum_{i,j=1}^{P}b_{ij}g_j\left(z_i(t-\tau(t))(1-\tau)\operatorname{sgn}(z_i)\right)-\sum_{i=1}^{P}\left(M_i\sum_{j=1}^{P}|b_{ji}|\right)\left|z_i(t-\tau(t))(1-\tau)\right|\\
&\le\sum_{i=1}^{P}M_i\sum_{j=1}^{P}\left[|b_{ij}||z_i|-\tau|b_{ji}||z_i(t-\tau(t))|-|z_i|\right]+\sum_{j=1}^{P}\left(a_{jj}+\sum_{1\le i\ne j\le P}|a_{ij}|\right)|g_j(z_j)|\\
&\le\sum_{i=1}^{P}M_i\sum_{j=1}^{P}\left[|b_{ij}||z_i|-\tau|b_{ji}||z_i(t-\tau(t))|-|z_i|\right]+\sum_{j=1}^{P}\left(a_{jj}+\sum_{1\le i\ne j\le P}|a_{ij}|\right)^{+}M_j|z_j|\\
&=\sum_{j=1}^{P}\sum_{i=1}^{P}\left[M_j\left(|b_{ij}|+\left(a_{ij}+\sum_{1\le i\ne j\le P}|a_{ij}|\right)^{+}-1\right)|z_j|-M_j\tau|b_{ij}||z_i(t-\tau(t))|\right]\\
&\le\sum_{j=1}^{P}\sum_{i=1}^{P}\left[M_j\left(|b_{ij}|-\tau|b_{ij}|-1+\left(a_{ij}+\sum_{1\le i\ne j\le P}|a_{ij}|\right)^{+}\right)\right]|z_j|.\end{aligned}$$

If

$$\max_{1\le j\le P}\left[M_j\left(\sum_{i=1}^{P}|b_{ij}|(1-\tau)+a_{ij}+\sum_{1\le i\ne j\le P}|a_{ij}|\right)\right]<1,$$

then $\dot V(z_t)\big|_{(6)}<0$. Thus, by the Lyapunov stability theorems [14], we can conclude that the unique equilibrium point x^* of system (6) is GAS. This completes the proof.

Corollary 1 If $M_j=1,\ j=1,2,\ldots,P$, and $\mu_1(A)+\|B\|_1<1$, where

$$\mu_1(A)=\max_{1\le j\le P}\left\{a_{jj}+\sum_{1\le i\ne j\le P}|a_{ij}|\right\}$$

is the measure of the matrix A and $\|B\|_1$ is the 1-norm of the matrix B, then the DCNN (7) is global asymptotic stability.

Remark1

In (6), assume $b_{ij}=0, M_j=1, (i,j=1,2,\cdots,P)$ that the DCNN is reduced to CNN. Then, the condition of theorem 1 is $\mu_1(A)<1$ which is much weaker than that of $\mu_1(A)<1$ in the literature[12].

Remark 2 The condition of theorem 1 is $\|A\|+\|B\|<1$ .In fact, $\forall A\in R^{P\times P}$, the condition in the literature [13] is $\mu(A)\le\|A\|$[13] that results $\mu(A)+\|B\|\le\|A\|+\|B\|$ .So, from $\|A\|+\|B\|<1$, we can get $\mu(A)+\|B\|\le 1$. Then, the conditions in the literature[13] are stronger than condition in this paper.

Next, in (6), denote $\tau(t)=0$, then (6) can be written

$$\dot z(t)=-z_i(t)+\sum_{j=1}^{P}a_{ij}g_j(z_j)+\sum_{j=1}^{P}b_{ij}g_j(z_j(t)),\ \ i=1,2,\cdots,P.$$

(7)

Assumed that $g_j(z_j)$ is piecewise linear function. Let $R = [r_{ij}]_{P\times P}$, where r_{ij} is defined

$$r_{ij} = \begin{cases} \left(a_{ij} + b_{ij}(1-\tau)\right)^{+} - 1, & i = j; \\ |a_{ij} + b_{ij}|, & i \neq j. \end{cases}$$

Theorem2 If the measure of matrix R is $\mu_2(R) = \lambda_{\max}\left(\frac{R+R^T}{2}\right) < 0$, then CNN (7) is globally asymptotically stable. Furthermore, there must be a positive number $\Delta > 0$ such that the delay cellular neural network (DCNN) (6) is also asymptotically stable.

Proof. Firstly, let

$$V(t) = \frac{1}{2}\sum_{i=1}^{P} z_i^2, \qquad (8)$$

where $V(t)$ is positive definite and radially unbounded.

$$\begin{aligned} \dot{V}(t)\Big|_{(8)} &= -\sum_{i=1}^{P} z_i^2 + \sum_{i,j=1}^{P} z_i\left(a_{ij}+b_{ij}\right)g_j(z_j) \\ &\le -\sum_{i=1}^{P} z_i^2 + \sum_{i=1}^{P}\left[z_i\left(a_{ii}+b_{ii}\right)g_i(z_i) + \sum_{j\neq i}|z_i||a_{ij}+b_{ij}||z_j|\right] \\ &\le \sum_{i=1}^{P}\left(\left[\left(a_{ii}+b_{ii}\right)^{+} - 1\right]z_i^2 + \sum_{j\neq i}|z_i||a_{ij}+b_{ij}||z_j|\right) \\ &= |z|^T R|z| \le \lambda_{\max}\left(\frac{R+R^T}{2}\right)z^T z. \end{aligned}$$

In the above, $|z| = \left(|z_1|,|z_2|,\ldots,|z_P|\right)^T$. Therefore, if $\mu_2(R) = \lambda_{\max}\left(\frac{R+R^T}{2}\right) < 0$, we can get $\dot{V}(z)\Big|_{(8)} < 0$. Then, the cellular neural network (7) is globally asymptotically stable.

Secondly, the delayed cellular neural network (DCNN) (6) can be written as

$$\dot{z}_i = -z_i + \sum_{j=1}^{P}\left(a_{ij}+b_{ij}\right)g_j(z_j) + \sum_{j=1}^{P} b_{ij}\left[g_j\left(z_j\left(t-\tau(t)\right)\left(1-\dot{\tau}(t)\right)\right) - g_j(z_j)\right]$$

.Defined $\frac{dy_i}{dx_i} = n_i(x_i) = \begin{cases} 1, \text{iff } |x_i| \le 1; \\ 0, \text{iff } |x_i| > 1. \end{cases}$ For the delayed cellular neural network (DCNN) (6), we use the above $V(t)$ as the Lyapunov function. When $\xi \in [t-\omega, t]$, by using the integral mean value theorem and the Razumikhin condition, we drive

$$\begin{aligned} \frac{dV}{dt}\Big|_{(6)} &= \frac{dV}{dt}\Big|_{(7)} + \sum_{i=1}^{P}\frac{\partial V}{\partial z_i}\left[\sum_{j=1}^{P} b_{ij}\left[g_j\left(z_j\left(t-\tau(t)\right)\left(1-\dot{\tau}(t)\right)\right) - g_j\left(z_j(t)\right)\right]\right] \\ &\le \frac{dV}{dt}\Big|_{(7)} + \sum_{i,j=1}^{P}|z_i||b_{ij}|\left|\int_{t-\tau(t)}^{t} n_j(z_j)\dot{z}_j(s_j)\,ds_j\right| \\ &\le \frac{dV}{dt}\Big|_{(7)} + \tau(t)\sum_{i,j=1}^{P}|z_i||b_{ij}|\left|-z_j(\xi_j) + \sum_{i=1}^{P}\left[a_{ji}g_i(\xi_i) + b_{ji}g_i\left(\xi-\tau(t)\right)\right]\right| \\ &\le \frac{dV}{dt}\Big|_{(7)} + \tau(t)\sum_{i,j=1}^{P}|z_i||b_{ij}|\left[|z_j(\xi_j)| + \sum_{i=1}^{P}\left(|a_{ji}||g_i(\xi_i)| + |b_{ji}||g_i\left(\xi-\tau(t)\right)|\right)\right] \\ &\le \frac{dV}{dt}\Big|_{(7)} + \omega\sum_{i,j=1}^{P}|z_i||b_{ij}|\left[\sum_{j=1}^{P}\left(|a_{ji}| + |b_{ji}| + 1\right)\right]|z_j|. \end{aligned}$$

Since the cellular neural network (CNN) (7) is asymptotically stable, there is $\frac{dV}{dt}\Big|_{(7)} < 0$.

Assumed that $\frac{dV}{dt}\Big|_{(7)} = -W(z_1, z_2, \cdots, z_n)$.

Obviously, the second terms is positive. Then, by denoting

$$\omega U(z_1, z_2, \cdots, z_n) = \omega\sum_{i,j=1}^{P}|z_i||b_{ij}|\left[\sum_{j=1}^{P}\left(|a_{ji}| + |b_{ji}| + 1\right)\right]|z_j|,$$

we can get

$$\frac{dV}{dt}\Big|_{(6)} \le -W(z_1, z_2, \cdots, z_n) + \omega U(z_1, z_2, \cdots, z_n).$$

Since $W(z_1, z_2, \cdots, z_n)$ is positive which implies $-W(z_1, z_2, \cdots, z_n)$ negative definite. Then, there is $m_1 > 0$ such that $-W \le -m_1 V$. Similarly, there is $m_2 > 0$ such that $U \le -m_2 V$. Hence, $\frac{dV}{dt}\Big|_{(6)} \le (\omega m_2 - m_1)V$. As long as there is $\omega \le \frac{m_1}{m_2} = \Delta$, it can be concluded $\frac{dV}{dt}\Big|_{(6)} \le 0$.

Thus, by the Lyapunov stability theorems [13], the delayed cellular neural network (DCNN) (6) is globally asymptotically stable. This completes the proof.

Illustrative Example

Now, we give two examples to present the merits of our result.

Example1. Consider the following delayed cellular neural network

$$A = \begin{bmatrix} -0.5 & 1 \\ 0 & -2 \end{bmatrix}, \quad B = \begin{bmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{bmatrix}, \qquad 9)$$

where the non diagonal elements of matrix A are positive or zero and all the elements of matrix B are positive or zero. By calculating, we can get

$$\max_{1\le j\le P}\left\{\left(\sum_{i=1}^{P}\left|b_{ij}(1-\tau)\right|+a_{jj}+\sum_{1\le i\ne j\le P}\left|a_{ij}\right|\right)\right\}=0.5(1-\tau)$$

.Then, according to theorem 1, the system (9) is globally asymptotically stable. Fig.1 depicts the time responses of state variables $x(t)$. It confirms that the proposed condition leads to the global asymptotic stability for the model.

Remark5. Denote matrix $-(A+B)$ is

$$-(A+B)=\begin{bmatrix}0 & -1.5\\ -0.5 & 1.5\end{bmatrix}.$$

Obviously, it is not row-dominant and the conditions in document [8] are not satisfied.

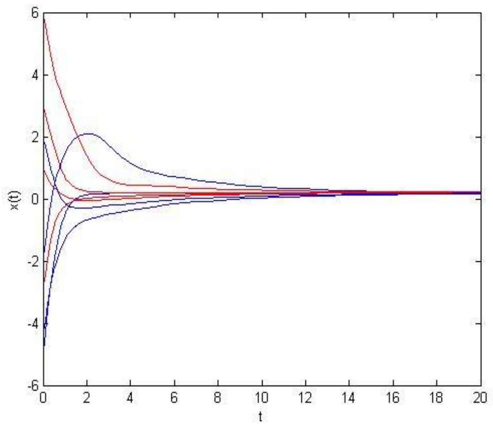

Figure1. *Stability regions in Example 1 with* $\tau=0.1$, $\tau(t)=e^{-t}$.

Example2. Consider the following system, where $A=\begin{bmatrix}-1 & 0.08\\ 0.9 & -0.09\end{bmatrix}$, $B=\begin{bmatrix}0.5 & 0.5\\ 0.5 & 0.5\end{bmatrix}$. (10)

By calculating, we can get $$\max_{1\le j\le P}\left\{\left(\sum_{i=1}^{P}\left|b_{ij}(1-\tau)\right|+a_{jj}+\sum_{1\le i\ne j\le P}\left|a_{ij}\right|\right)\right\}=0.9(1-\tau)$$

. Then, according to theorem 1, the system (10) is globally asymptotically stable. Since matrix $-(A+A^T)=\begin{bmatrix}2 & -0.98\\ -0.9 & 0.18\end{bmatrix}$ is not positive definite, the conditions in document [8] are not satisfied. Fig.2 depicts the time responses of state variables $x(t)$. It confirms that the proposed condition leads to the global asymptotic stability for the model.

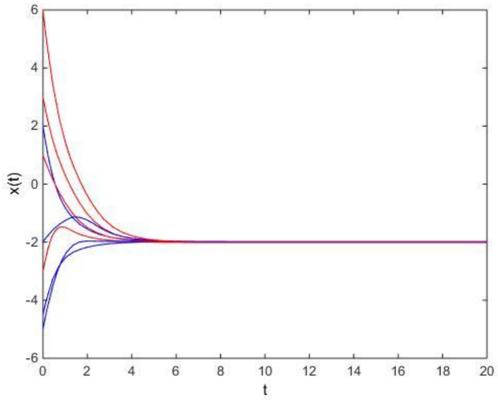

Figure2. *Stability regions in Example 2 with* $\tau=0.02$, $\tau(t)=e^{-t}$

CONCLUSION

This paper has discussed the global asymptotic stability for the Cellular neural networks with time-varying delays. By constructing suitable Lyapunov functional, some sufficient conditions are obtained without demanding the differentiability of the activation functions. Two simulation examples are shown the effectiveness of the proposed method.

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