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Solution of Fuzzy Transportation Problem with Elimination Method

Poonam Shugani
MathematicsDepartment,
G.D.Rungta college of engineering and technology,
Bhilai,(C.G.),India

S. H. Abbas
Mathematics Department,
Saifia Science college,
Bhopal (M.P.), India

Vijay Gupta
Mathematics Department,
UIT RGPV,
Bhopal (M.P.), India

Abstract: *The object of the fuzzy transportation problem is to determine the shipping schedule that minimizes the total fuzzy transportation cost while satisfying fuzzy supply and demand limits. A fuzzy transportation problem basically deals with the problem, which aims to find the best way to fulfill the demand of n demand points using the capacities of m supply points. Here we studied a new method for solving fuzzy transportation problems with mixed constraints and find an optimal solution. The optimal solution procedure is illustrated with numerical example. Though maximum fuzzy transportation problems in real life have mixed constraints, these problems are not be solved by using general method. The proposed method builds on the initial solution of the Fuzzy transportation problem which is very simple, easy to understand and apply.*

Keywords: *Fuzzy transportation problem, mixed constraints, elimination method, triangular numbers*.

1. Introduction

Transportation problems are one of the powerful frame works which ensures efficient movement and timely availability of the raw materials and finished goods. This transportation problem is a linear programming problem obtained from a network structure consisting of a defined numbers of nodes and arcs attached to them. Let us consider in which a production is to be transported from m sources to n destinations and their capacities $a_1, a_2, \cdots, a_m$ and $b_1, b_2, \cdots, b_n$ respectively. In addition to this there is a penalty C_{ij} associated with transporting unit of production from source i to destination j this penalty may be either cost or delivery time or safety delivery. A variable x_{ij} represents the unknown quantity to be shipped from source i destination j.

Lot of efficient algorithms had been developed for solving the transportation problems when the cost coefficients and the supply and demand quantities are known exactly. However, there are cases that these parameters may not be presented in an exact manner. For example the unit shipping cost may vary in a time frame. The supplies and demands may be uncertain due to some uncontrollable factors. To deal quantitatively with not exact information in making decisions, Bellman and Zadeh[4] introduce the notion of fuzziness. Chanas S., Kolodziejczyk W. and Machaj A.[6] have given concept about fuzzy approach to the transportation problem. Since the transportation problem is almost a linear programme one straight forward idea is to apply the existing fuzzy linear programming techniques to the fuzzy transportation problems.

Unfortunately most of the existing techniques only provide crisp solutions. The method of Julien and Parraet[8] al is able to find the possibility distribution of the objective values provided all the inequality constraints are of " $\leq$ " type or " $\geq$ " type. However due to the structure of the transportation problem in some cases, their method requires the refinement of the problem parameters to be able to derive bounce of the objective value. There are also some studies discussing the fuzzy transportation problem. Chanas et al [1,2,4,9,13], investigates the transportation problems with fuzzy supplies and demands and solve them via the parametric programming techniques in terms of Bellman-Zadhe[4] criterion. Their method is to derive the solution which simultaneously satisfies the constraints and the goal to a maximal degree. Soniya and Rita malhotra[14] discribe a two stage minimizing transportation problem. Adlakha V. and Kowalski K.[3] and Chanas A. and Klingman D.[7] Pandian P. and Natarajan, G. [11,12] has given mor for less solution about fuzzy transportation problem.

Chanas and Kuchta[5] discuss the type the transportation problems with fuzzy cost coefficients and transforms the problem to bicriterial transportation with crisp objective function. Their method is able to determine the efficient solutions of the transformed problem, unless only crisp solutions are provided. Pandian P. and Natarajan, G.[10] has given some algorithms for finding optimal solution of fuzzy transportation problem.

This paper finds the best compromise solution among the set of feasible solution for the fuzzy transportation problem. We are using elimination method for removing some variables from equations then applying fourier elimination method for optimal solution. To illustrate the proposed method, some examples is used. Finally some conclusions are obtained from the discussion.

2. Fuzzy Transportation Problem with Mixed Constraints

Considering the mathematical model for a fuzzy transportation problem with mixed constraints

Minimize

$$Z = \sum_{i=1}^{n} \sum_{j=1}^{n} \tilde{c}_{ij} x_{ij}$$

Subject to

$$\sum_{j=1}^{n} x_{ij} \geq \tilde{a}_i \qquad i \in Q$$

$$\sum_{j=1}^{n} x_{ij} \leq \tilde{a}_i \qquad i \in T$$

$$\sum_{j=1}^{n} x_{ij} = \tilde{a}_i \qquad i \in S$$

$$\sum_{i=1}^{m} x_{ij} \geq \tilde{b}_j \qquad j \in U$$

$$\sum_{i=1}^{m} x_{ij} \leq \tilde{b}_j \qquad j \in v$$

$$\sum_{i=1}^{m} x_{ij} = \tilde{b}_j \qquad j \in w$$

$x_{ij} \geq 0, i = 1,2,3, \dots \dots m \text{ and } j = 1,2,3 \dots \dots n \text{ and integers}$

where Q,T and S are pair wise disjoint subsets of { 1,2,3,...,n }

Such that $Q \cup T \cup S$ = {1,2,3,...,n };

U, V and W are pair wise disjoint subsets of{ 1,2,3,...,m }

Such that $U \cup V \cup W = \{1,2,3,\dots,m\}$;

c_{ij} is the cost of shipping one unit from supply point i to the demand point j ;

a_i is the supply at supply point i ;b_j is the demand at demand point j and

x_{ij} is the number of units shipped from supply point i to demand point j.

3. Elimination Method

We are using elimination method for finding an optimal solution of different type of fuzzy transportation problems of mixed constraints of triangular and trapezoidal numbers also.

The elimination method proceeds as follows.

1. Write the given fuzzy transportation problem of trapezoidal or triangular numbers with mixed constraints.
2. Write the problem with mixed constraints in the form of a pure integer linear programming problem.
3. Convert the pure integer linear programming problem into a maximization problem.
4. Convert the equations of maximization problem in the type of the inequality $\leq$ by eliminating one variable in an equality constraint of the problem.
5. Write the equivalent pure integer linear programming problem to the modified maximization problem and then, construct the elimination table for the equivalent problem.
6. Select and remove a variable from the elimination table.
7. Form the last reduced elimination table get transportation cost.

4. Numerical Example

Considering the following balanced fuzzy transportation problem of triangular numbers:

	D_1	D_2	D_3	Supply
S_1	5	9	13	(20,50,80)
S_2	11	18	20	(30,35,40)
S_3	14	13	16	(30,40,50)
Demand	(10,30, 50)	(20,40, 60)	(35,55, 75)	

Now, the pure integer linear programming problem of above problem is given by

Maximize

$Z=5x_{11}+9x_{12}+13x_{13}+11x_{21}+18x_{22}+20x_{23}+14x_{31}+13x_{32}+16x_{33}$

Subject to

$X_{11}+X_{12}+X_{13} = (20,50,80)$

$X_{21}+X_{22}+X_{23} = (30,35,40)$

$X_{31}+X_{32}+X_{33} = (30,40,50)$

$X_{11}+X_{21}+X_{31} = (10,30,50)$

$X_{12}+X_{22}+X_{32} = (20,40,60)$

$X_{13}+X_{23}+X_{33} = (35,55,75)$

$X_{11},X_{12},X_{13},X_{21},X_{22},X_{23},X_{31},X_{32},X_{33} \geq 0$ and integers.

And the minimization problem is

$W = -5x_{11}-9x_{12}-13x_{13}-11x_{21}-18x_{22}-20x_{23}-14x_{31}-13x_{32}-16x_{33}$

After elimination of X_{11}, $X_{12},X_{13},X_{21},X_{31}$ the min. equation will become

$W = (-2780,-1795,-810) -16X_{22}-X_{23}+18X_{32}+6X_{33}$

Subject to $X_{22}+X_{23} \leq (30,35,40)$

$X_{32}+X_{33} \leq (30,40,50)$

$X_{22}+X_{32} \leq (20,40,60)$

$X_{23}+X_{33} \leq (35,55,75)$

Then maximize

$-16X_{22} - X_{23}+18X_{32}+6X_{33}+W \leq (-2780,-1795,-810)$

Then $X_{32} = 0$ and $X_{33} =0$

Then $X_{22},X_{23},X_{32},X_{33} \geq 0$

Now we have $X_{32} = 0$ and $X_{33} = 0$.

Then $-16X_{22} - X_{23}+W \leq (-2780,-1795,-810)$

$X_{32} \leq (30,40,50)$

$X_{33} \leq (35,55,75)$

$X_{22}+X_{23} \leq (30,35,40)$

$X_{22}, X_{23} \geq 0.$

Now the elimination table is

X_{22}	X_{23}	W	B
-16	-1	1	(-2780,-1795,-810)
1	0	0	(30,40,50)
0	1	0	(35,55,75)
1	1	0	(30,35,40)
-1	0	0	(0,0,0)
0	-1	0	(0,0,0)

After elimination of X_{22} and X_{23}

W	B
1	(-2300,-1155,-10)

Hence W = (-2300,-1155,-10)

And Z = (10,1155,2300)

5. Numerical Example 2: An Unbalanced Fuzzy Transportation Problem

	D_1	D_2	D_3	Supply
S_1	9	2	5	(2,5,8)
S_2	3	6	4	(3,6,9)
S_3	8	9	2	(3,9,15)
Demand	(4,8,12)	(8,10,12)	(3,5,7)	

Maximization

$Z=9X_{11}+2X_{12}+5X_{13}+3X_{21}+6X_{22}+4X_{23}+8X_{31}+9X_{32}+2X_{33}$

Subject to

X11+X12+X13 = (2,5,8)

X21+X22+X23 $\geq$ (3,6,9)

X31+X32+X33 $\leq$ (3,9,15)

X11+X21+X31 = (4,8,12)

X12+X22+X32 $\geq$ (8,10,12)

X13+X23+X33 $\leq$(3,5,7)

Minimize $W = -(25,38,51)-9X_{11}-5X_{13}-X_{22}-X_{23}-8X_{31}-9X_{32}-2X_{33}$

$X_{11} + X_{13} \leq (2,5,8)$

$X_{31} + X_{32} + X_{33} \leq (3,9,15)$

$X_{13} + X_{23} + X_{33} \leq (3,5,7)$

$X_{11} + X_{31} \leq (4,8,12)$

$X_{11} + X_{13} - X_{22} - X_{32} \leq (-10,-5,0)$

$X_{11} + X_{31} - X_{22} - X_{23} \leq (-5,2,9)$

$X_{11}, X_{13}, X_{22}, X_{23}, X_{31}, X_{32}$ and $X_{33} \geq 0$ and integers

If all integers $X_{11}, X_{13}, X_{22}, X_{23}, X_{31}, X_{32}$ and $X_{33} = 0$ and integers.

Now the equivalent problem to the problem is given below:

Maximize

$W=9X_{11}+5X_{13}+X_{22}+X_{23}+8X_{31}+9X_{32}+2X_{33} + W \leq -(25,38,51)$

Subject to

$X_{11} + X_{13} \leq (2,5,8)$

$X_{31} + X_{32} + X_{33} \leq (3,9,15)$

$X_{13} + X_{23} + X_{33} \leq (3,5,7)$

$X_{11} + X_{31} \leq (4,8,12)$

$X_{11} + X_{13} - X_{22} - X_{32} \leq (-10,-5,0)$

$X_{11} + X_{31} - X_{22} - X_{23} \leq (-5,2,9)$

$X_{11} = 0$, $X_{13} = 0$, $X_{31} = 0$, $X_{23} = 0$ and $X_{33} = 0$

Then

$X_{22} + 9X_{32} + W \leq -(25, 38, 51)$

$X_{32} \leq (3, 9, 15)$

$-X_{22} - X_{32} \leq (-10,-5, 0)$

$-X_{22} \leq (-5, 2, 9)$

$X_{22}, X_{32} \geq 0$ and integers.

The elimination table is

X_{22}	X_{32}	W	B
1	9	1	-(25,38,51)
0	1	0	(3,9,15)
-1	-1	0	(-10,-5,0)
-1	0	0	(0,0,0)
0	-1	0	(0,0,0)

After elimination of X_{22} and X_{32}

W = - (52,119,152)

i.e. Z = (52,119,152)

Hence the transportation cost is (52,119,152)

6. Conclusion

We have attempt a new elimination method to find an optimal solution of Fuzzy Transportation Problems with mixed constraints. The proposed method for an optimal solution is very simple, easy to understand and apply. The analysis could be useful for managers in making strategic decisions such as increasing aware-house stocking level or plant production capacity and advertising efforts to increase demand at certain markets. So, the new method for an optimal solution using elimination method can serve managers by providing one of the best optimal solutions to a variety of distribution problems.

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